Wong's equations and the small x effective action in QCD [*]

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One of the more interesting open questions in QCD is the behavior of cross–sections at very high energies. In the last decade, a kinematic window has opened up at colliders where $Q^2 \gg \Lambda^2_{QCD}$ but $x = Q^2/s \ll 1$. The physics in this regime is non–perturbative because the field strengths at small x are large. However, it is also weak coupling physics since $\alpha_S(Q^2) \ll 1$. Further, since the density of partons is large at small x, classical field methods are applicable.

Such an effective field theory approach has been used to study the physics of small x modes in QCD by McLerran and Venugopalan. The small x effective action is obtained by successively integrating out the more static modes at larger values of x. This effective action approach also reproduces the standard linear evolution equations of perturbative QCD in the limit of low parton densities. However, the truly interesting and unknown regime is the non–linear regime of high parton densities where one might hope to predict novel phenomena. What the correct effective action is in the high density regime should therefore be a matter of some interest.

In a paper publishe in Physical Review D[1], my collaborators Jalilian-Marian and Venugopalan and I proposed an alternative gauge invariant form to the original small x action proposed by McLerran and Venugopalan. Motivation for this form of the effective action came from our recent work in formulating a many body world line formalism for the one loop effective action in QCD written previously by us. Briefly, the difference between the two actions is in the term describing the coupling of the small x gauge field modes to the large x modes represented by a color charge density ρ. In the work of Jalilian–Marian, Kovner, Leonidov, and Weigert (JKLW), this term is expressed as

$$S^{JKLW}_{int} \sim \mathrm{Tr}\,(\rho\, W_{\infty,-\infty})\ ,$$

where W is an adjoint matrix corresponding to a path ordered exponential of the gauge field A^- in the light cone direction x^+. We propose instead that this term be

$$S_{int} \sim \mathrm{Tr}\,(\rho\, \ln W_{\infty,-\infty})\ ,$$

replacing $W \to \ln W$ in the effective action.

This change from W to $\ln W$ can be understood as follows. Using the background field method, and the Eikonal approximation, the one loop effective action is $\ln[\det(D^2)] \to \mathrm{Tr}\ln[D^2]$, where D is the usual covariant derivative. In the world line formalism, the $\mathrm{Tr}\ln(D^2)$ is re–written as a quantum mechanical path integral over world lines. Alternately, in the Eikonal approach, one anticipates a path ordered phase from the operator. We showed that S_{int}, unlike S^{JKLW}_{int}, is consistent with Wong's equations that describe the motion of a classical colored particle in a background gauge field. In addition, we also showed that one obtains the BFKL equation in a more efficient manner using this effective action. This form of the effective action is therefore likely be useful in computing contributions beyond the leading $\alpha_S \log(1/x)$ term in the high parton density regime.

Footnotes and References

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[1] Phys. Rev. D63:036004, 2001